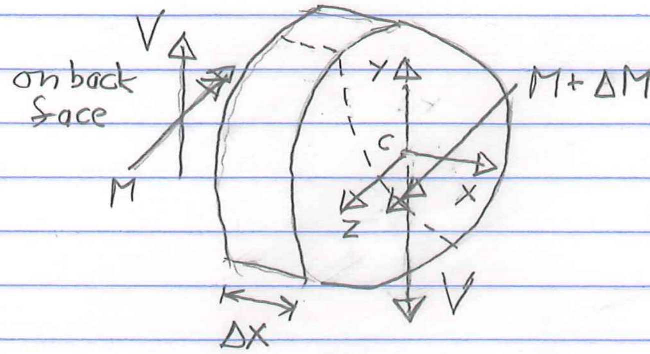
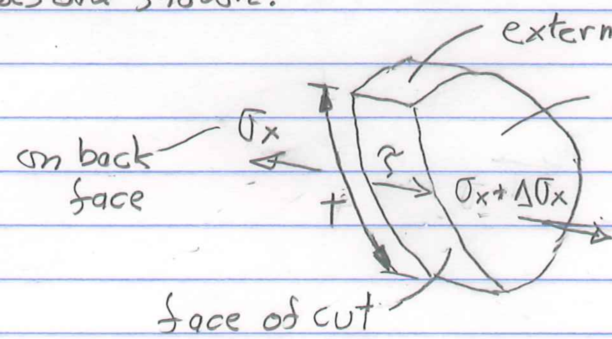
22. Shear flow and average shear stress
(Linearly elastic, homogeneous, isotropic material.



This shows a free body of a slice of the beam of length ΔX. Axis of M is parallel to z. x-y-z are principal, centroidal axes.

Examine a free body of the part of the slice beyond the cut indicated by the dashed line above. Equilibrium in the x direction is of interest, so only x direction stresses are shown.



$\Sigma F_x = 0$

$\int_{b-c} \Delta \sigma_x \, dA + \tau_{av} \cdot t \cdot \Delta X = 0$ where τ_{av} is the average of τ (the x-direction shear stress along the cut)

Take the limit as $\Delta X \to 0$ and substitute the following:

$$\frac{d\sigma_x}{dx} = -\frac{y}{I}\frac{dM}{dx} = -\frac{y}{I}V \quad \left(\text{Recall: } \sigma_x = -\frac{y}{I}M \text{ and } \frac{dM}{dx} = V\right)$$

We also used $\frac{dI}{dx} = 0$, which is justified for a uniform or slowly varying cross-section.

$\tau_{av} = \frac{q}{t}$, where q is the shear flow on the cut (shear force per unit length in the x direction).

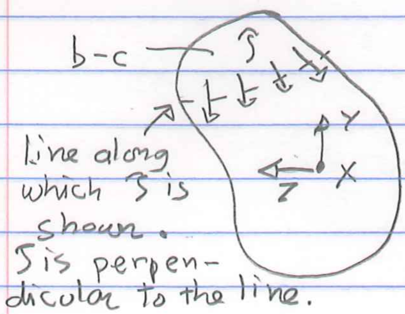
After substitution, the result is

$$q = \frac{VQ}{I} \quad \text{where} \quad Q = \int_{b-c} y \, dA$$

Or τ_{av} can be computed directly as

$$\tau_{av} = \frac{q}{t} = \frac{VQ}{It}$$

Note that the shear stresses τ on the cut are accompanied by shear stresses on the cross-section, as shown below. Thus, we now have a way to compute shear stress on a cross-section (actually, the average shear stress along and perpendicular to a line that passes over the cross-section from one edge to another.).



The average of τ is $\tau_{av} = \frac{VQ}{It}$

where $Q = \int_{b-c} y \, dA$ and t is the length of the line shown. X-Y-Z are principal, centroidal axes.

Without solving the equations of elasticity, we can't do any better than approximating the shear stress τ along and perpendicular to a line (that passes over the cross-section from one edge to another) as constant at τ_{av}. However, for many applications, τ is approximately constant, and τ_{av} is then an accurate result.